Introduction to low-level measurement techniques using short segments of conducting wire

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UNC-P

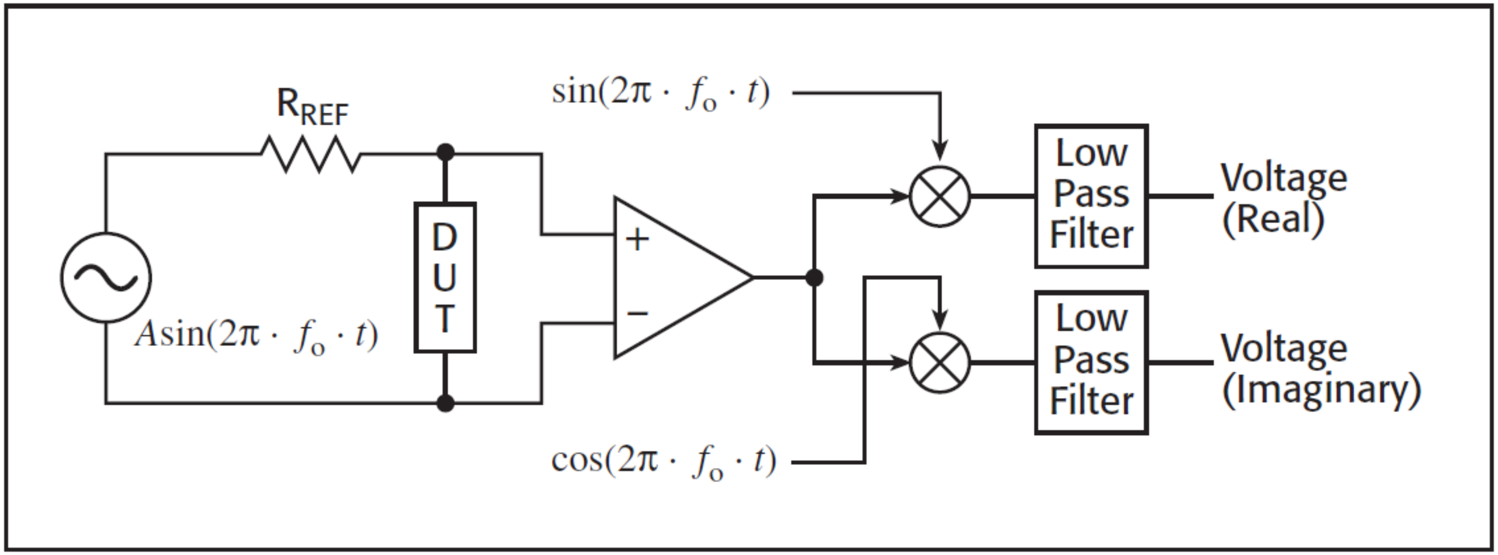
UNCP physics students investigated short segments (length ~ 10-45 cm, resistance ~ 33-45 mΩ) of copper wire (AWG 30, 24,22) to integrate information using an exhaustive list of activities.

- Effects of 60 Hz and 120 Hz notch filters of the SR 830 lock-in amplifier (refutation of Keithley "white paper")
- Calculation of resistance ($\rho l/a$) with wire diameter measured using diffraction pattern
- AC voltage characterization using SR 830 lock-in amplifier (refutation of a previous AJP article)

Cannot be trusted for high accuracy constraints

AWG	Diameter		Turns of wire, no insulation		Area		Copper resistance[6]	
	(in)	(mm)	(per in)	(per cm)	(kcmil)	(mm^2)	(Ω/km) (mΩ/m)	(Ω/kft) (mΩ/ft)
21	0.0285	0.723	35.1	13.8	0.810	0.410	42.00	12.80
22	0.0253	0.644	39.5	15.5	0.642	0.326	52.96	16.14
23	0.0226	0.573	44.3	17.4	0.509	0.258	66.79	20.36
24	0.0201	0.511	49.7	19.6	0.404	0.205	84.22	25.67
25	0.0179	0.455	55.9	22.0	0.320	0.162	106.2	32.37
26	0.0159	0.405	62.7	24.7	0.254	0.129	133.9	40.81
27	0.0142	0.361	70.4	27.7	0.202	0.102	168.9	51.47
28	0.0126	0.321	79.1	31.1	0.160	0.0810	212.9	64.90
29	0.0113	0.286	88.8	35.0	0.127	0.0642	268.5	81.84
30	0.0100	0.255	99.7	39.3	0.101	0.0509	338.6	103.2
31	0.00893	0.227	112	44.1	0.0797	0.0404	426.9	130.1

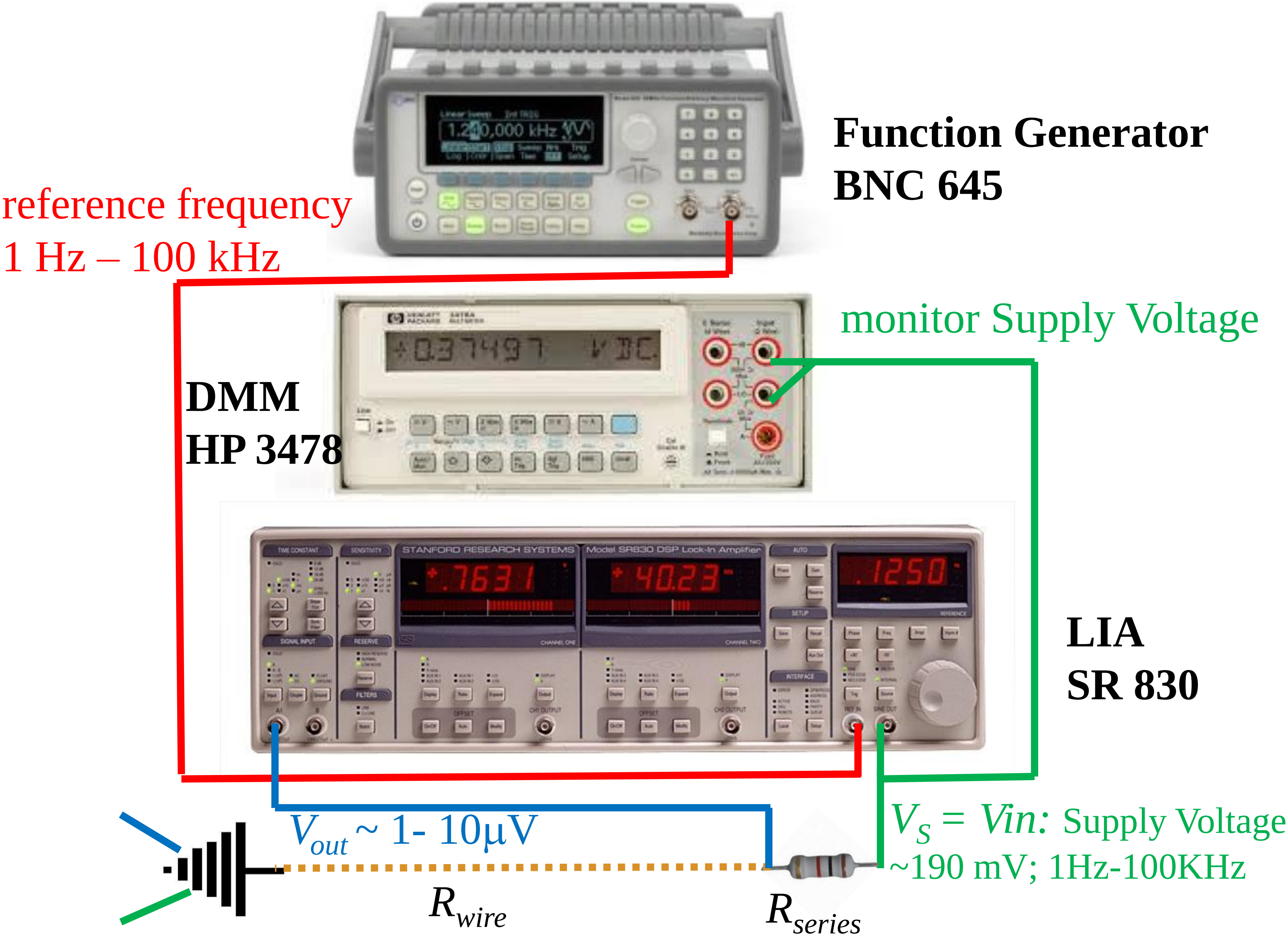
Simplified block diagram of a lock-in amplifier setup to measure the voltage of a DUT (e.g. a wire) at low power.



The amplified voltage from the DUT is multiplied by both a sine and a cosine wave with the same frequency and phase as the applied source and then subject to a low pass filter. In most cases, the multiplication and filtering is performed digitally within the lock-in amplifier after the DUT signal is digitized.

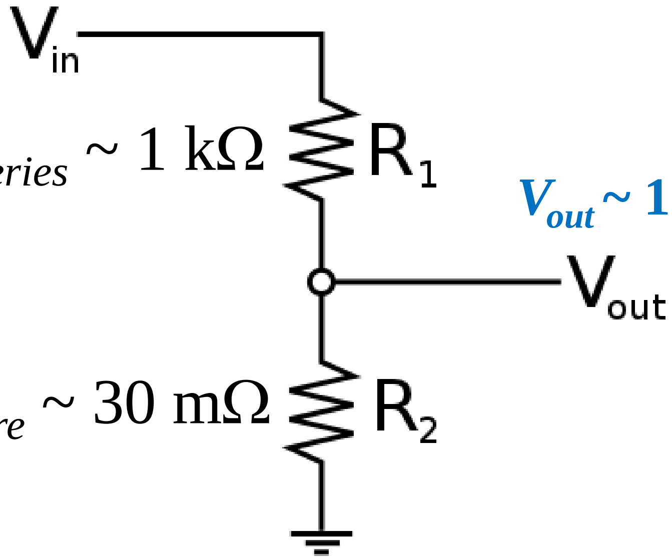
Extraction of signals buried in noise

Our latest method of choice:



Voltage Divider

V_{in} =190 mV; 1Hz-100KHz



$R_{series} \sim 1\ \text{k}\Omega$

$V_{out} \sim$ 1-10μV

$R_{wire} \sim 30\ \text{m}\Omega$

$$\boxed{V_{out} = \frac{R_{wire}}{R_{series} + R_{wire}}}$$

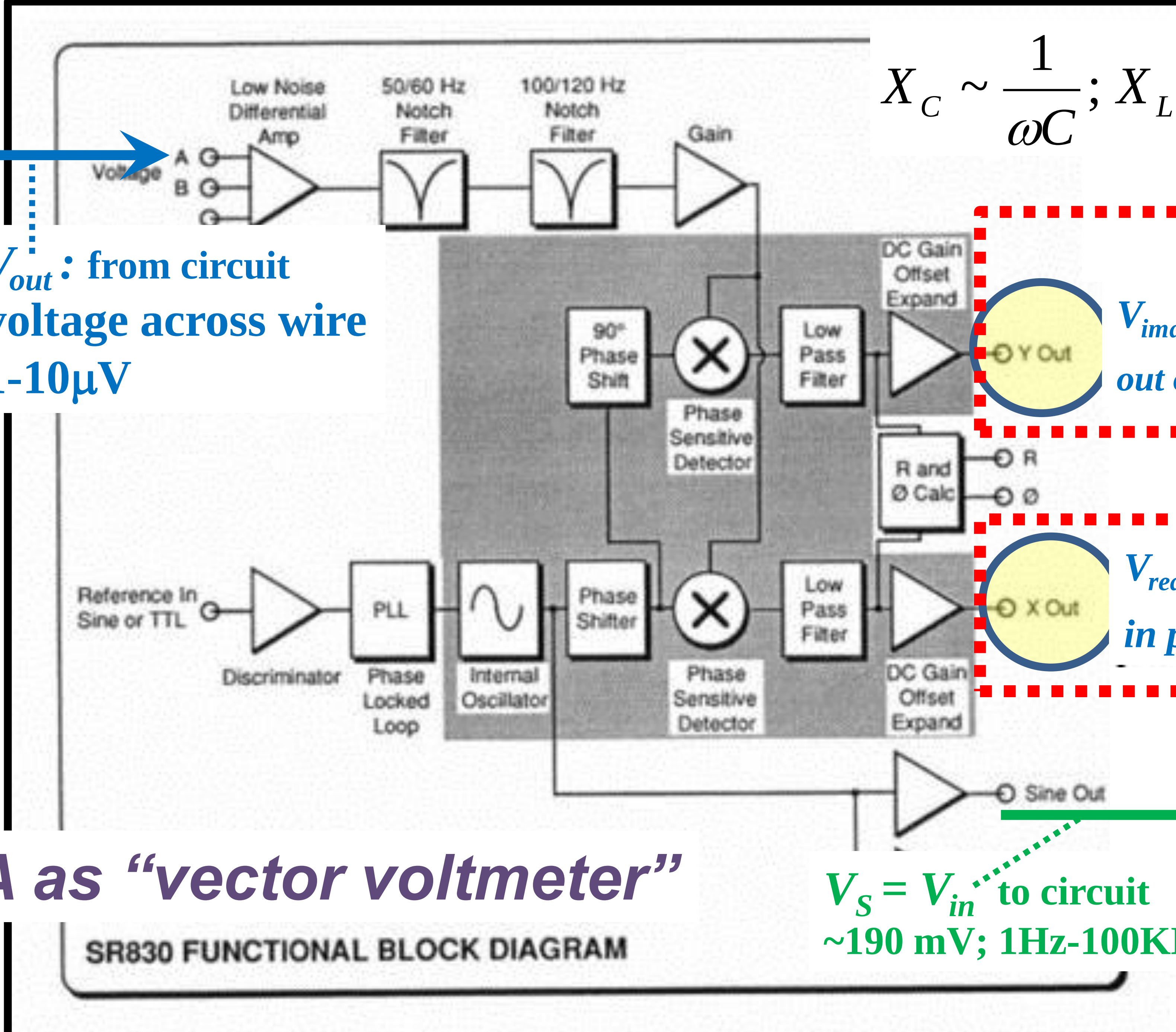
$$\left(R_{series} + R_{wire}\right) \approx R_{series}$$

$$R_{wire} \approx \frac{V_{out}}{V_{in}} R_{series}$$

our case: $R_{series} \approx 995\,\Omega\,; \quad V_{in} \approx 190\ mV$

$$R_{wire} \approx 5237 \times V_{out}$$

$$X_C \sim \frac{1}{\omega C}; \; X_L \sim \omega L$$



V_{out} : from circuit

voltage across wire

1-10μV

$V_{imaginary}$

out of phase

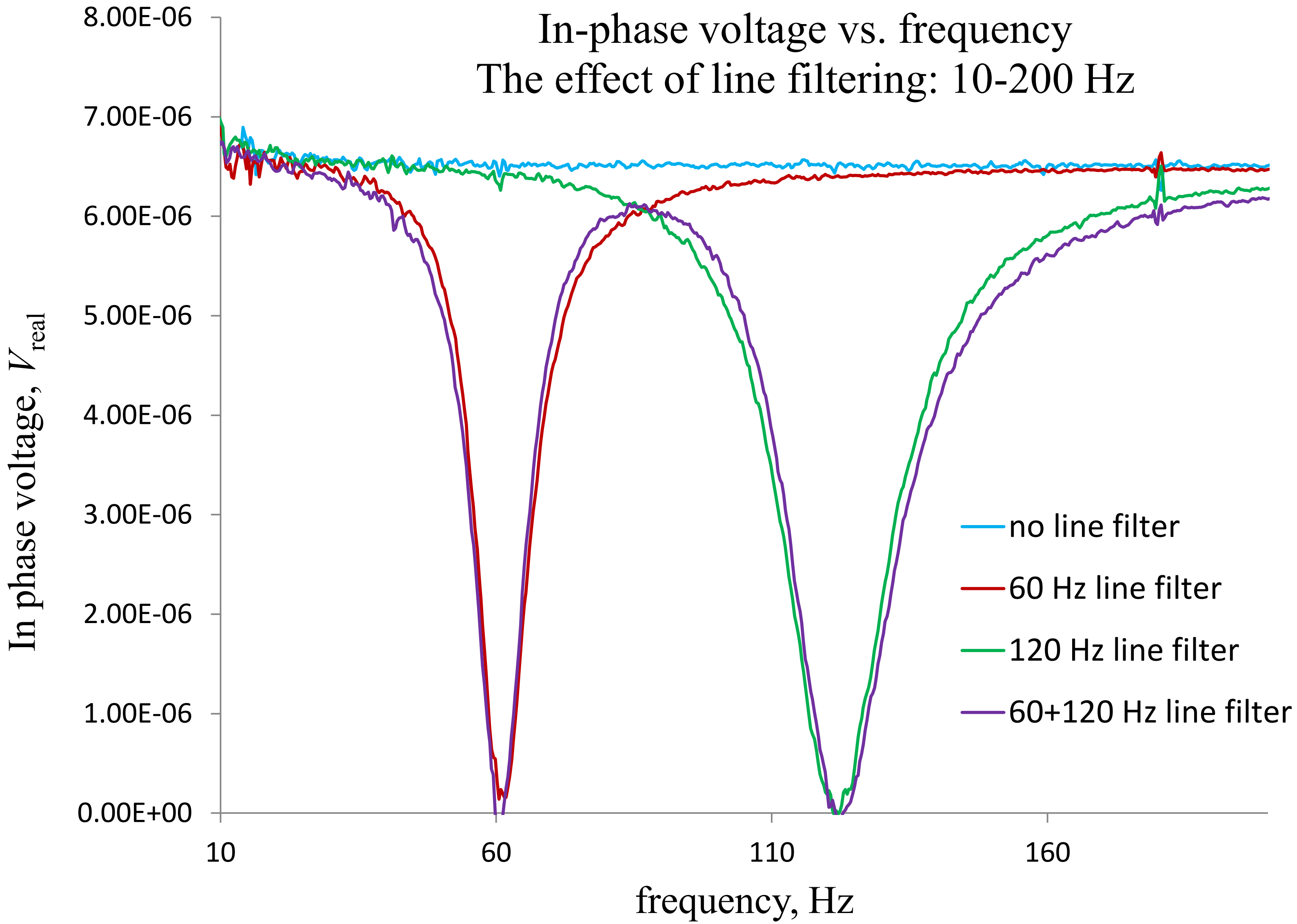
V_{real}

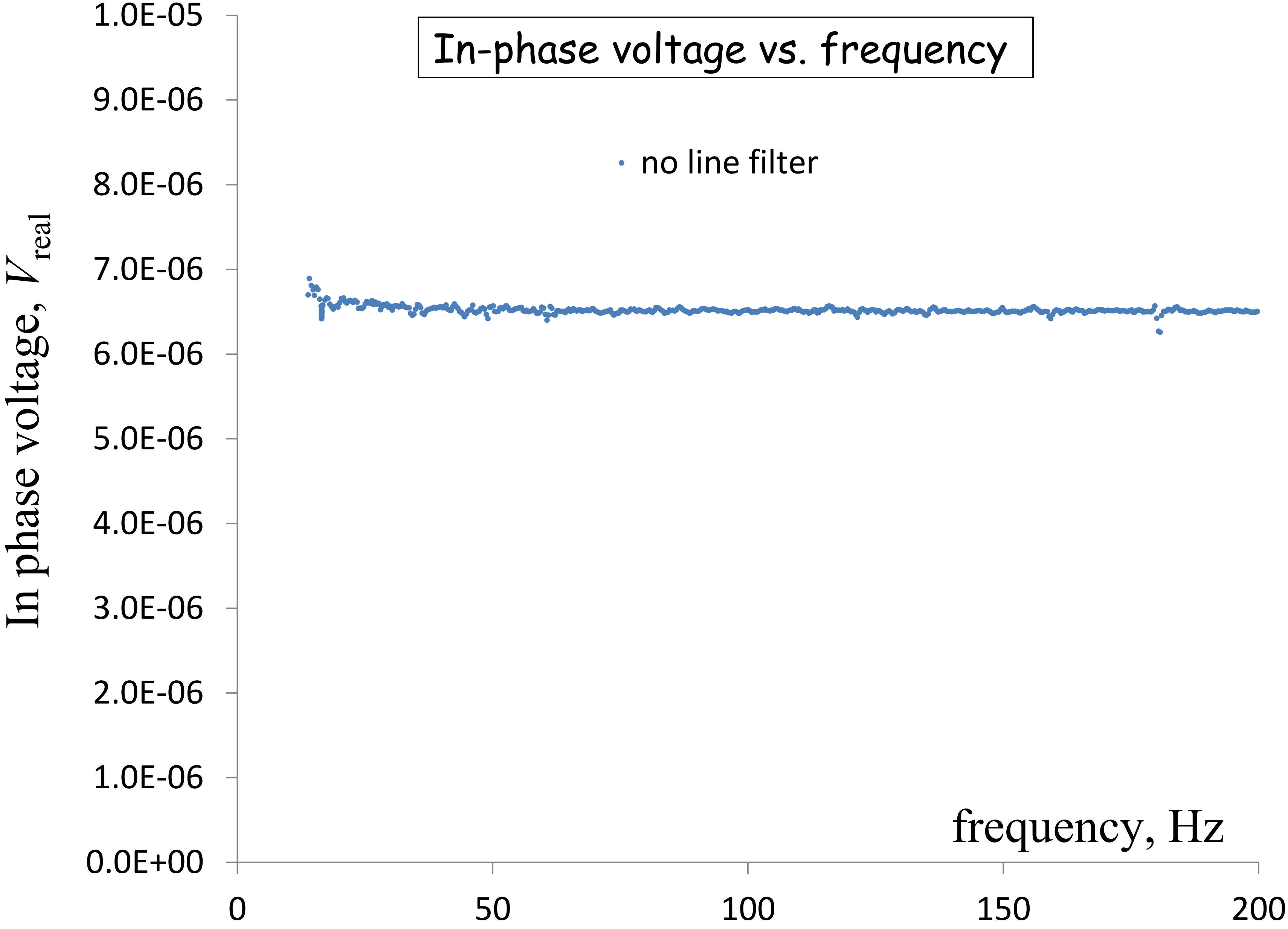
in phase

LIA as "vector voltmeter"

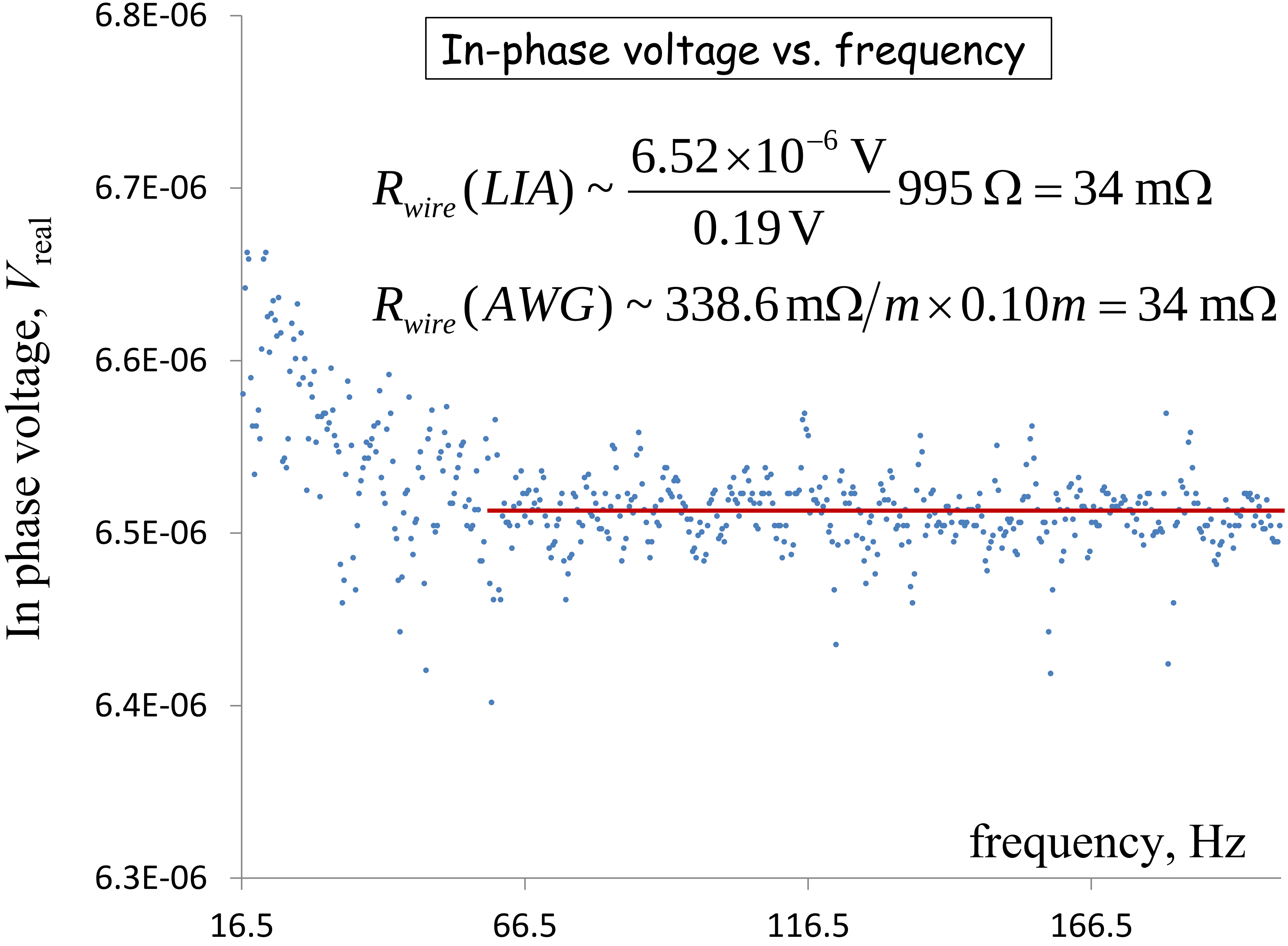
$V_S = V_{in}$ to circuit

~190 mV; 1Hz-100KHz


In-phase voltage vs. frequency
The effect of line filtering: 10-200 Hz
8.00E-06
7.00E-06
6.00E-06
5.00E-06
4.00E-06
3.00E-06
2.00E-06
1.00E-06
0.00E+00
10
60
110
160
frequency, Hz
In phase voltage, V_{real}
no line filter
60 Hz line filter
120 Hz line filter
60+120 Hz line filter

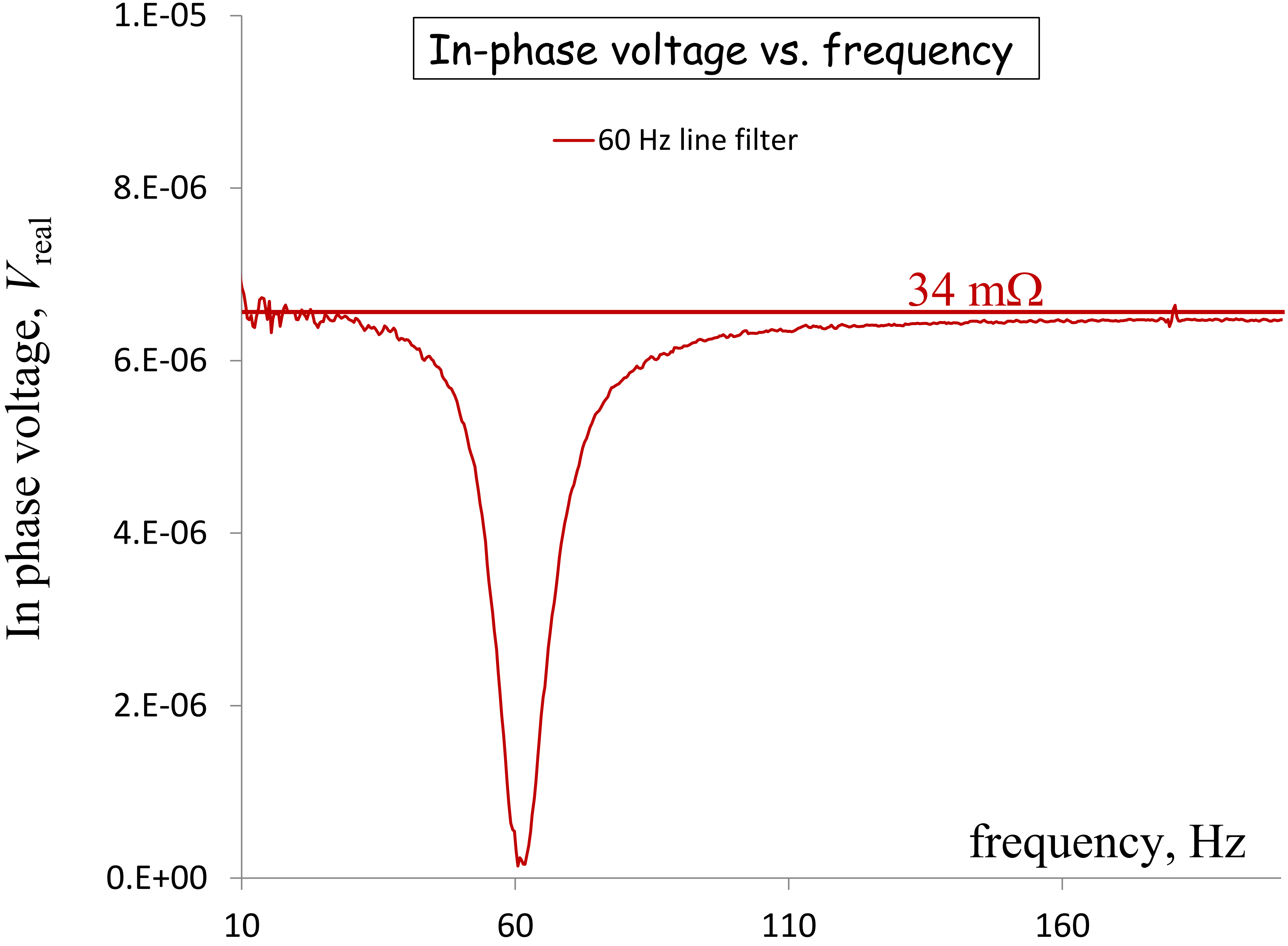


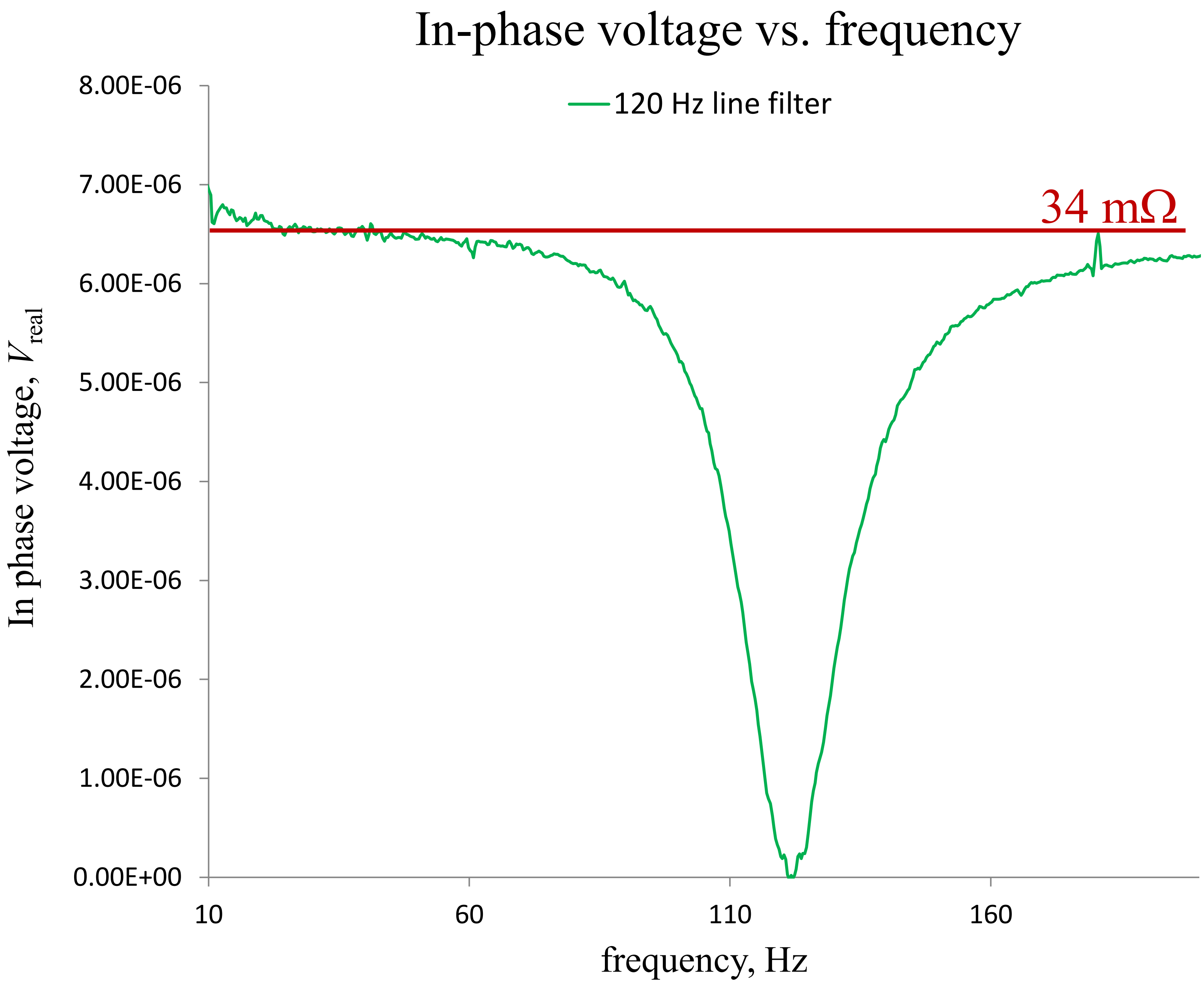
In-phase voltage vs. frequency
no line filter
In phase voltage, V_{real}
frequency, Hz
1.0E-05
9.0E-06
8.0E-06
7.0E-06
6.0E-06
5.0E-06
4.0E-06
3.0E-06
2.0E-06
1.0E-06
0.0E+00
0
50
100
150
200

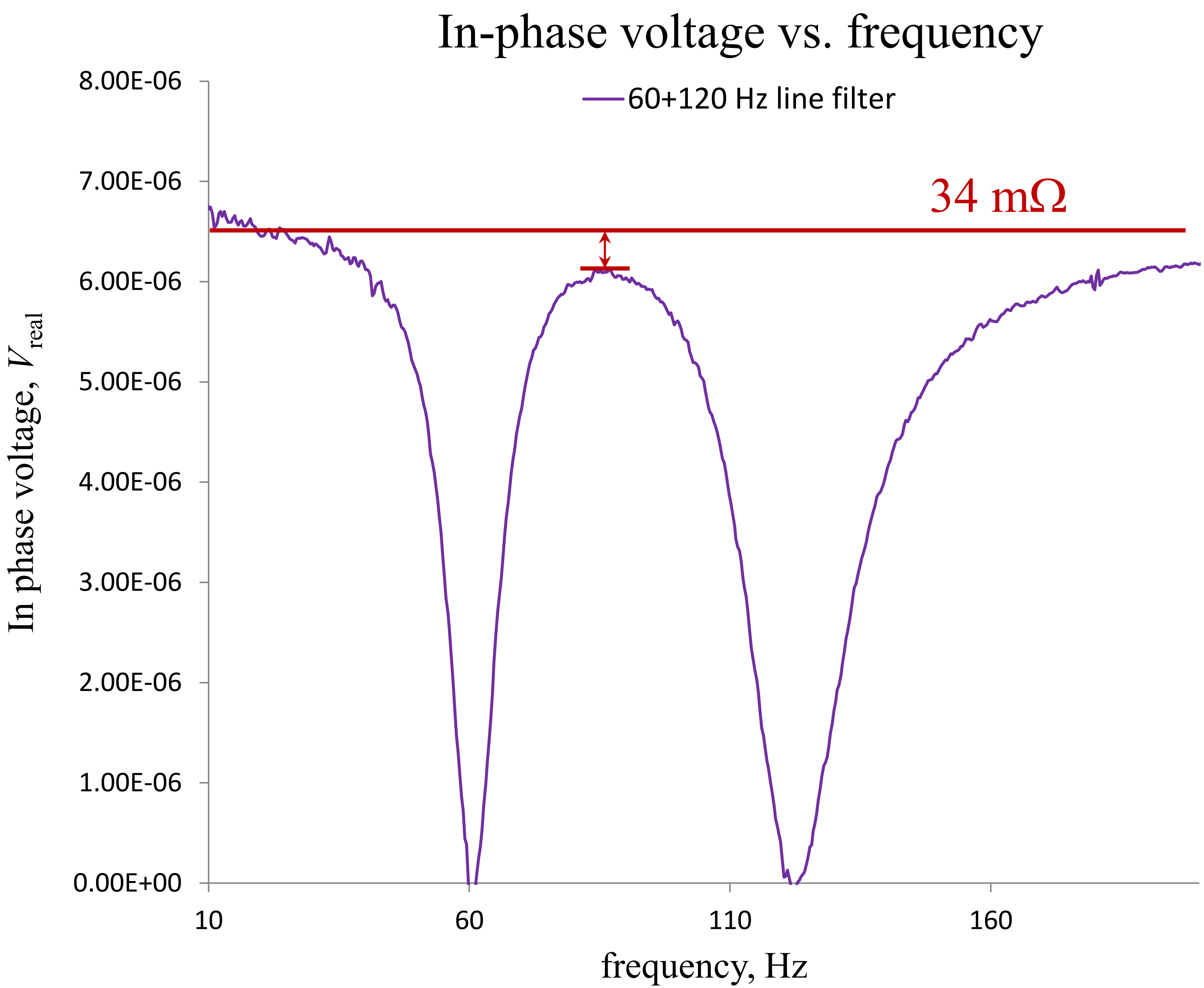


$$R_{wire}(LIA) \sim \frac{6.52\times 10^{-6}\ \mathrm{V}}{0.19\ \mathrm{V}} 995\ \Omega = 34\ \mathrm{m}\Omega$$

$$R_{wire}(AWG) \sim 338.6\ \mathrm{m}\Omega/m \times 0.10m = 34\ \mathrm{m}\Omega$$

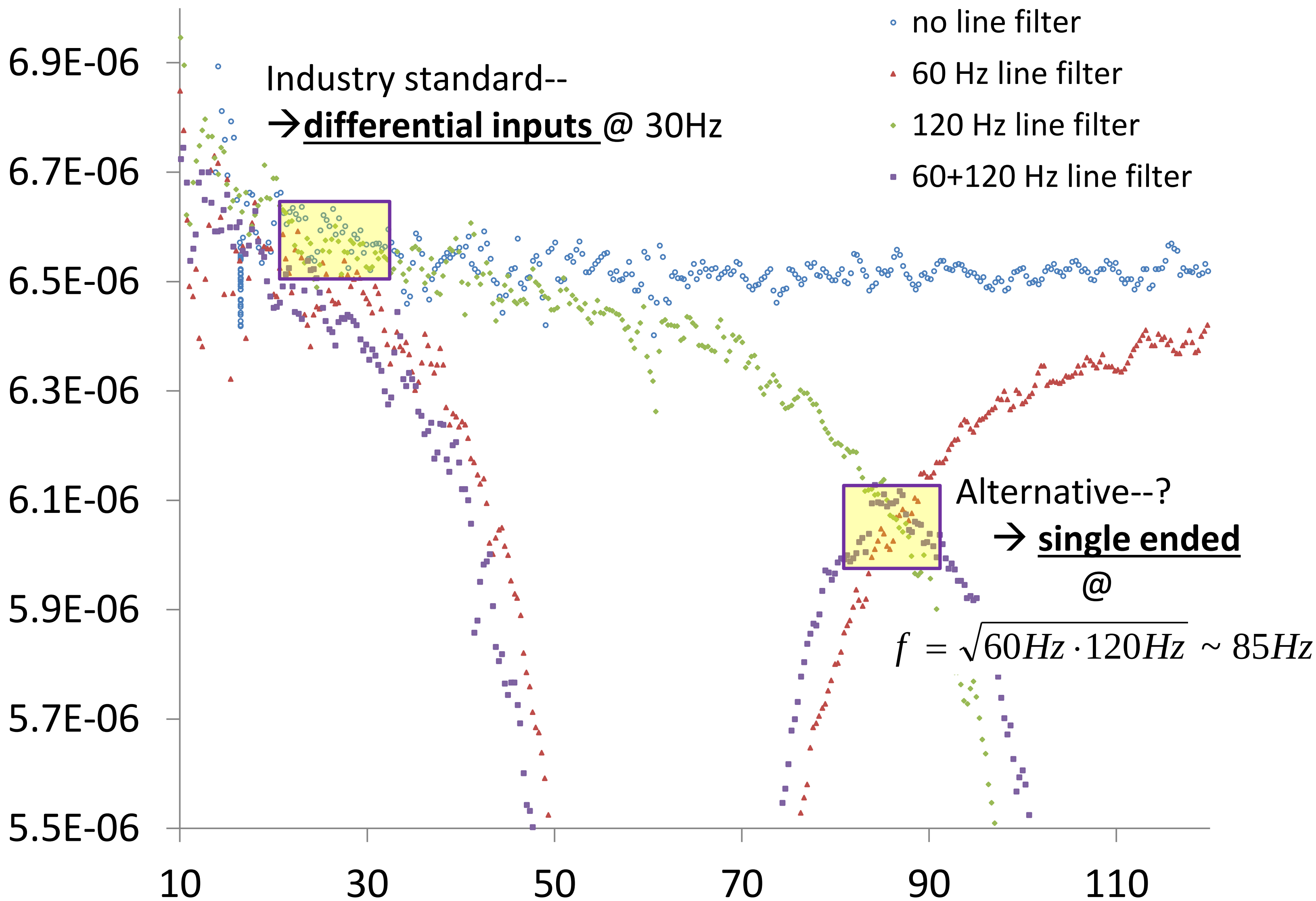

In-phase voltage vs. frequency
60 Hz line filter
34 mΩ
In phase voltage, V_{real}
frequency, Hz
1.E-05
8.E-06
6.E-06
4.E-06
2.E-06
0.E+00
10
60
110
160


In-phase voltage vs. frequency
120 Hz line filter
34 mΩ
8.00E-06
7.00E-06
6.00E-06
5.00E-06
4.00E-06
3.00E-06
2.00E-06
1.00E-06
0.00E+00
10
60
110
160
frequency, Hz
In phase voltage, V_{real}

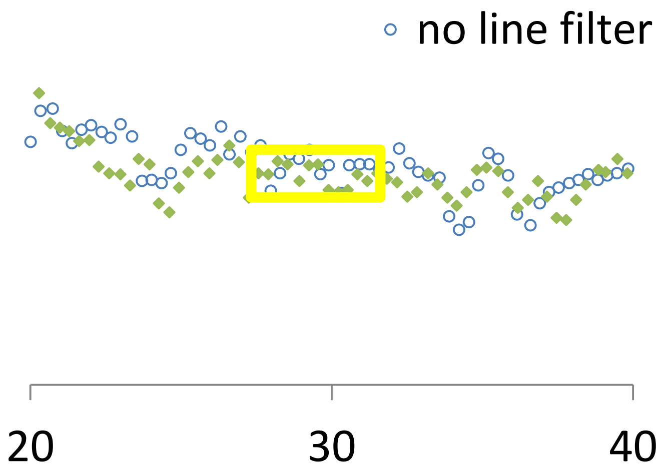


In-phase voltage vs. frequency
60+120 Hz line filter
34 mΩ
8.00E-06
7.00E-06
6.00E-06
5.00E-06
4.00E-06
3.00E-06
2.00E-06
1.00E-06
0.00E+00
10
60
110
160
In phase voltage, V real
frequency, Hz

In-phase voltage vs. frequency "choosing a frequency"



~30 Hz zoom



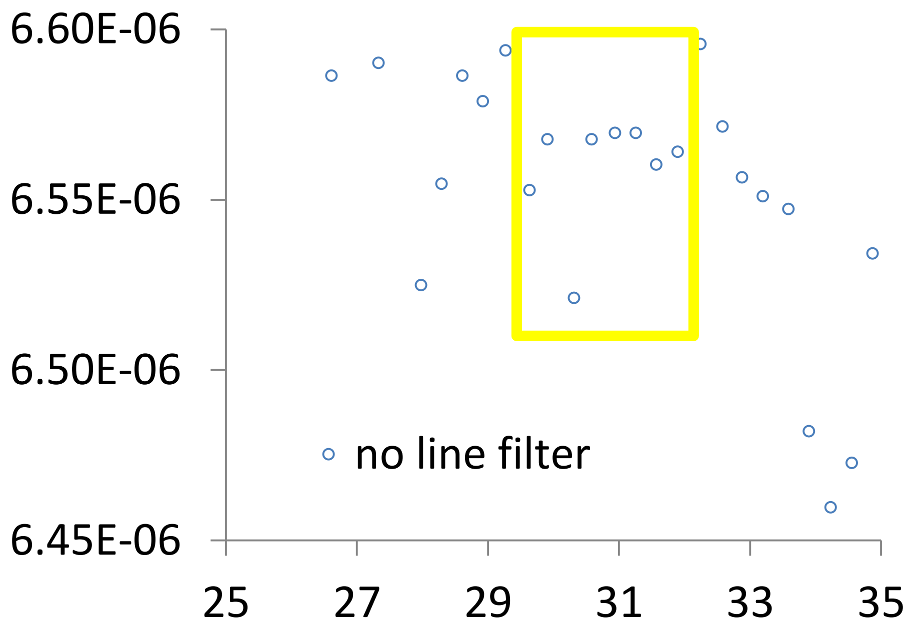
precision, in this case

Better than 1% **accuracy** (0.5 sec constraint)

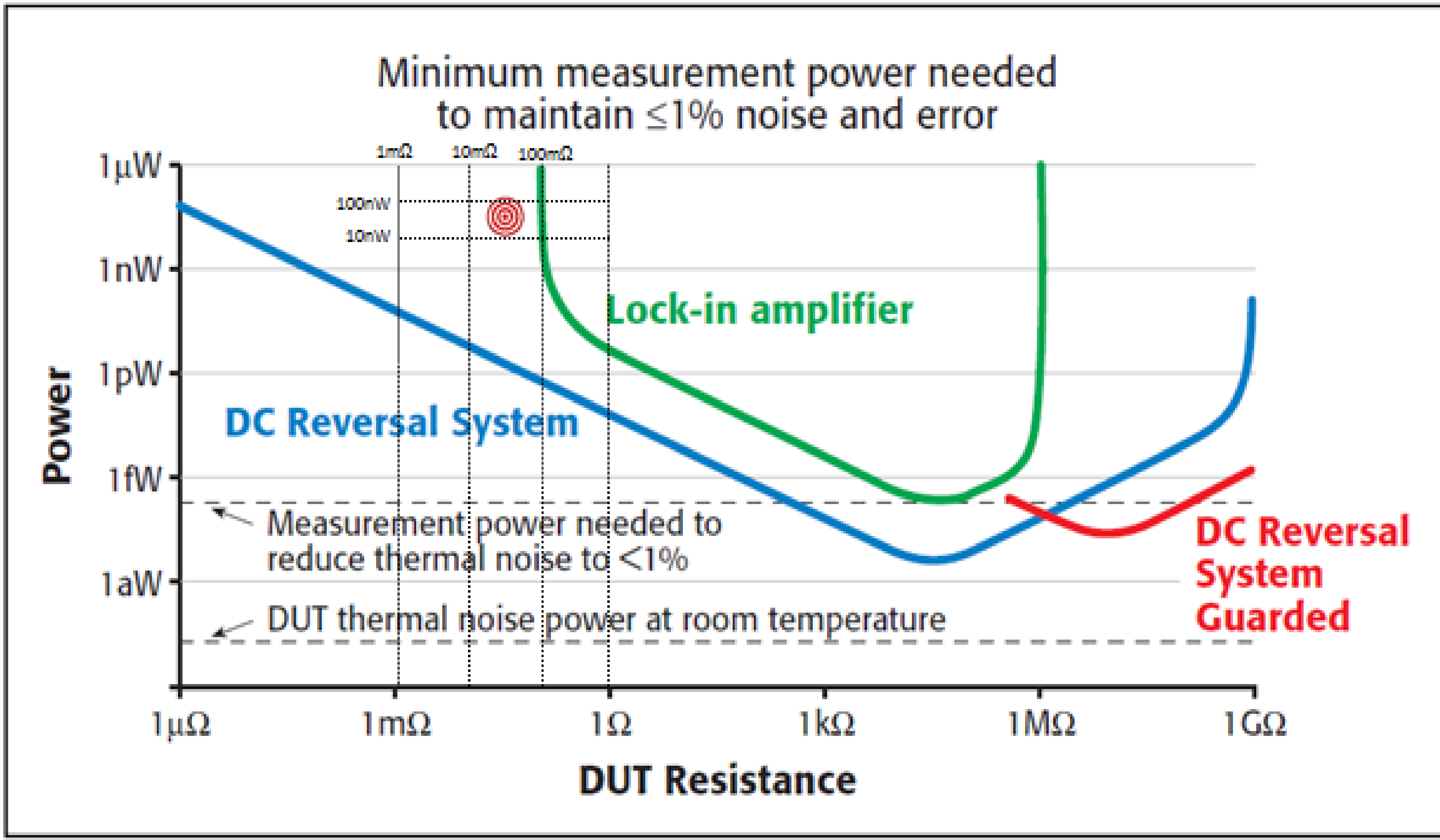
Suspect Claim 1**:** ***Keithley*** **white paper**

Tupta, "AC versus DC Measurement Methods for Low-power Nanotech and Other Sensitive Devices", 2007

Industry standard: use **differential inputs** @ 30Hz



Tupta, "AC versus DC Measurement Methods for Low-power Nanotech and Other Sensitive Devices", 2007



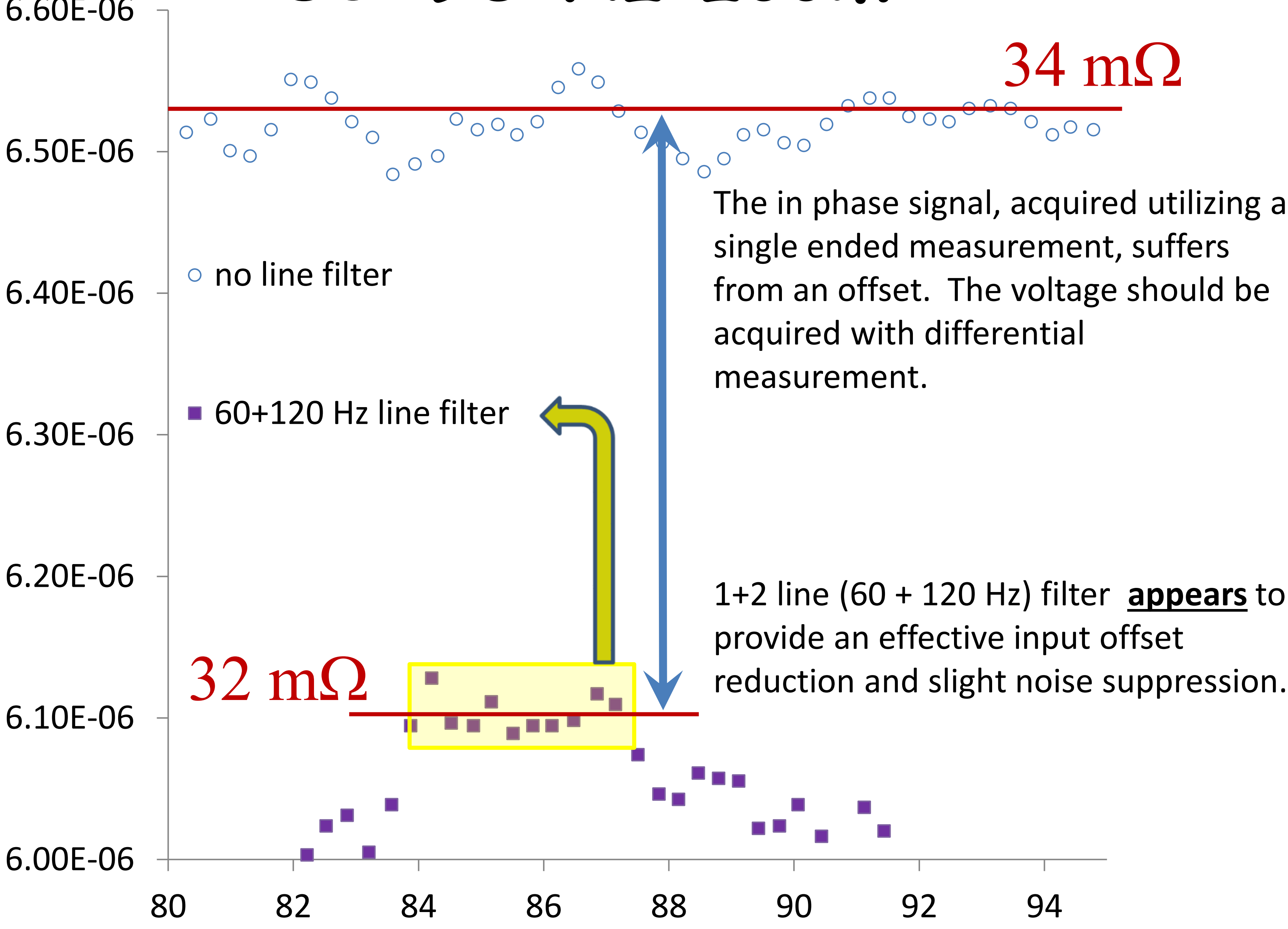
Next few slides:

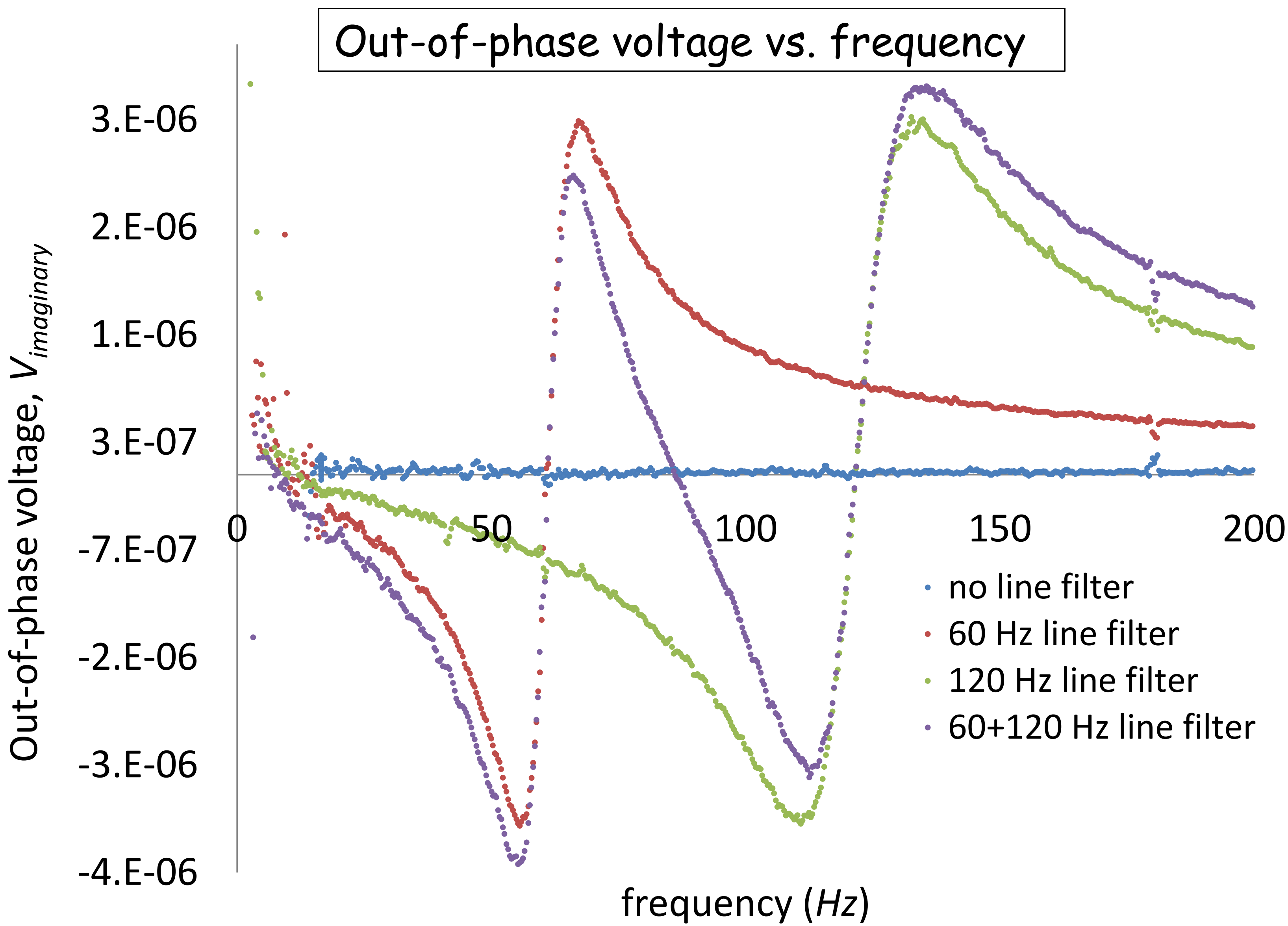
Lock-in configuraiton

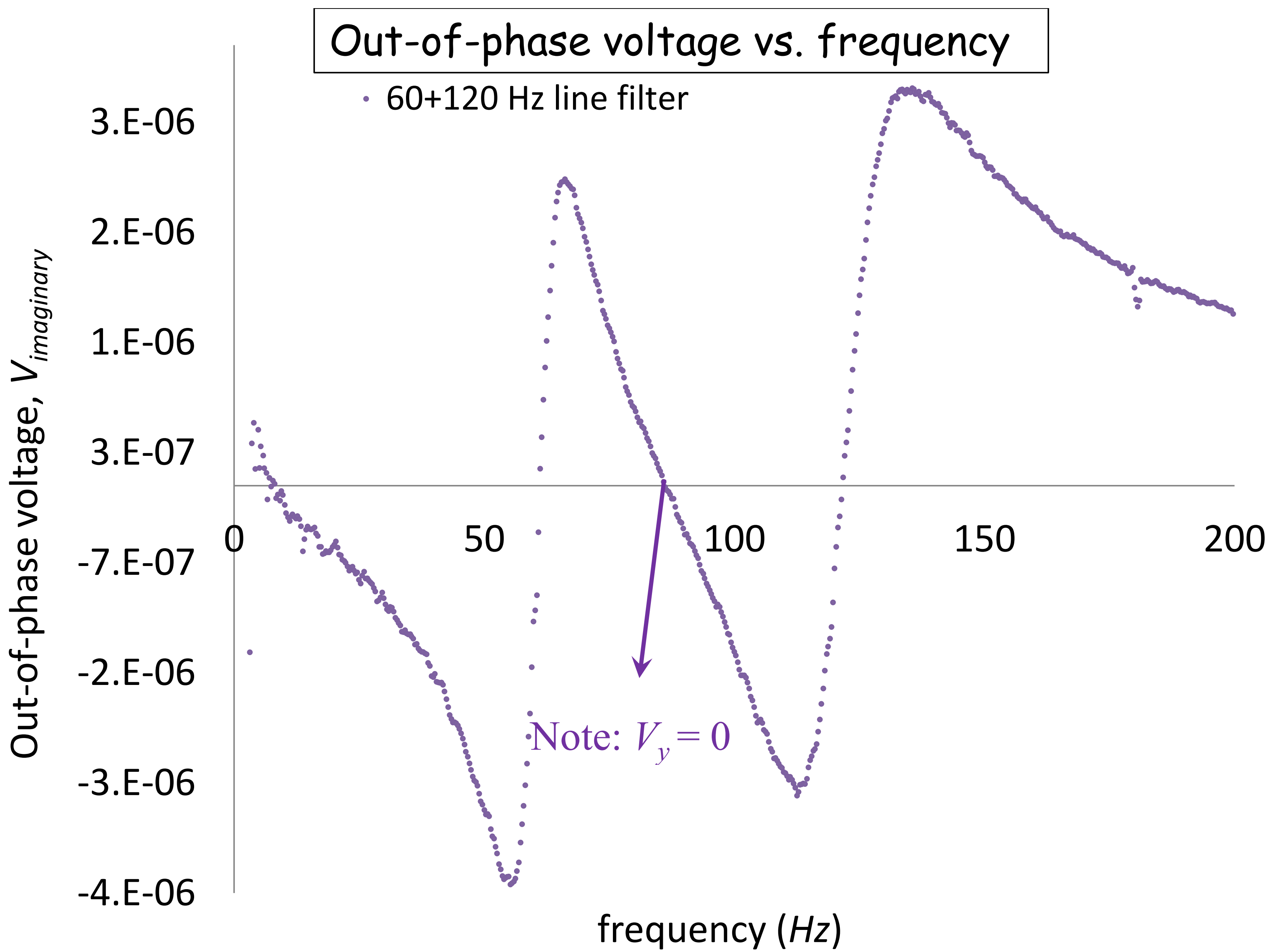
- 1f+2f line filters
- “Sync” function
- Fixed frequency of 85 Hz

… perhaps a novel –
and potentially useful approach?

~80-95 Hz zoom

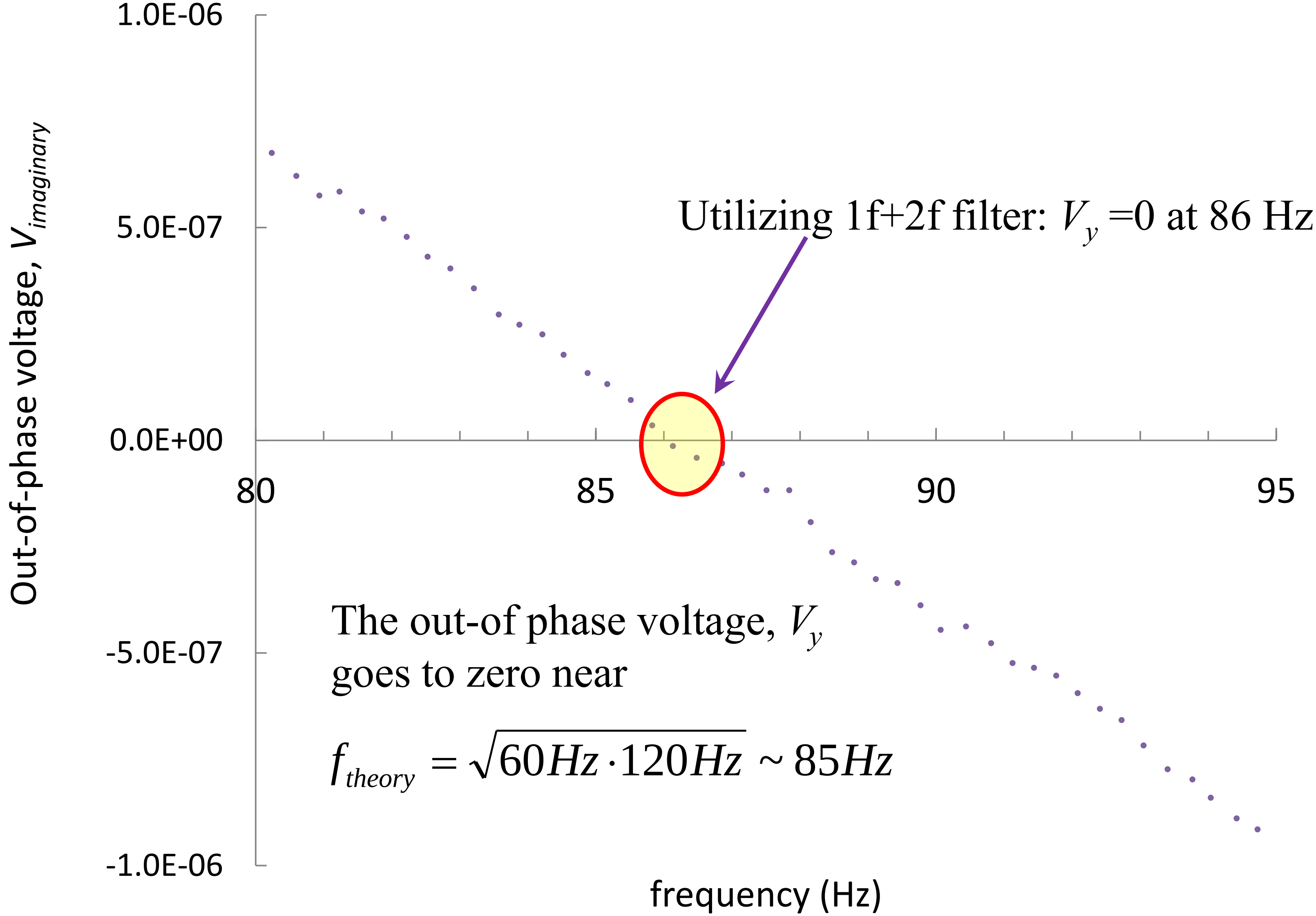

6.60E-06
6.50E-06
6.40E-06
6.30E-06
6.20E-06
6.10E-06
6.00E-06
80
82
84
86
88
90
92
94
34 mΩ
32 mΩ
no line filter
60+120 Hz line filter
The in phase signal, acquired utilizing a single ended measurement, suffers from an offset. The voltage should be acquired with differential measurement.
1+2 line (60 + 120 Hz) filter **appears** to provide an effective input offset reduction and slight noise suppression.


Out-of-phase voltage vs. frequency
Out-of-phase voltage, $V_{imaginary}$
3.E-06
2.E-06
1.E-06
3.E-07
-7.E-07
-2.E-06
-3.E-06
-4.E-06
0
50
100
150
200
frequency (Hz)
no line filter
60 Hz line filter
120 Hz line filter
60+120 Hz line filter



Out-of-phase voltage vs. frequency
60+120 Hz line filter
3.E-06
2.E-06
1.E-06
3.E-07
-7.E-07
-2.E-06
-3.E-06
-4.E-06
0
50
100
150
200
Out-of-phase voltage, $V_{imaginary}$
frequency (Hz)
Note: $V_y = 0$

Explanation of offset reduction



PHY 4200 final results of 1f+2f measurements at 85 kHz

Includes comparison to theoretical result.

internal ref	31.77$\pm$0.03	mΩ
external ref	31.81$\pm$0.07	mΩ
$R = \rho\, l\, /a$	31.6$\pm$0.8	mΩ

"expected"

Recall, from AWG table:

$$R_{wire}(AWG) \sim 338.6\,\mathrm{m\Omega}/m \times 0.10m = 34\ \mathrm{m\Omega}$$

How the “expected” result was obtained…

The expected resistance, R of a copper wire is known to obey the relation:

$$R = \rho \frac{l}{a}$$

$\Rightarrow \rho$: resitivity; l : length ; a : cross - sectional area

The resistivity, ρ is actually temperature, T dependent –

$$\rho(T) = \rho_0\left(1 + \alpha\left(T - T_0\right)\right)$$

$$\Rightarrow \rho_0 = 1.678 \times 10^{-8}\,\Omega m;\ \alpha = 0.003862/^{o}C; T_0 = 20^{0}C$$

$$\rho(T = 20.5 \pm 0.2) = 1.682 \times 10^{-8}\ \Omega m \qquad \Rightarrow \text{room temp} \pm \text{drift}$$

$$\rho(T = 20.5) = 1.6815(15) \times 10^{-8}\ \Omega m \qquad \Rightarrow \text{resistivity}$$

The cross sectional area, *a* was calculated from measurements using optical interferometry. The relation characterizing the **fringe pattern generated by diffraction** when a laser beam, with wavelength, λ, is obstructed by a wire with diameter, *d* is given by:

$d = \lambda \frac{L}{y}$ L: distance to fringe pattern y: mean fringe spacing

$$a = \pi d^2 / 4$$

$$\delta a = 2a\sqrt{\left(\frac{\delta\lambda}{\lambda}\right)^2 + \left(\frac{\delta L}{L}\right)^2 + \left(\frac{\delta y}{y}\right)^2} \approx 5.E^{-4}$$

Length, L	fringe spacing, y	diameter, d	area, a
mm	mm	mm	mm^2
3344(3)	8.13(5)	0.2605(16)	0.0533(5)
3344(3)	8.08(5)	0.2621(16)	0.0539(5)
3344(3)	8.13(5)	0.2603(16)	0.0532(5)
3344(3)	8.19(5)	0.2581(16)	0.0523(5)

mean	0.2603	0.0532
stdevp (σ)	0.0014	0.0006
prop unc (δ)	0.0016	0.0005

Measured at 4 points along the length of wire
~ 50 fringes

RESULT	$a = \pi d^2/4$	
$a(\delta)$ - statistics	0.532(6)	mm^2
$a(\delta)$ - estimated	0.535(5)	mm^2

"Expected" result

$$R = \rho \frac{l}{a}\left(1 \pm \underbrace{\sqrt{\left(\frac{\delta\rho}{\rho}\right) + \left(\frac{\delta l}{l}\right) + \left(\frac{\delta a}{a}\right)}}_{\text{propagted uncertainty}}\right) = 31.6 \pm 0.8\ \mathrm{m}\Omega$$

internal ref	31.77±0.03	mΩ
external ref	31.81±0.07	mΩ
$R = \rho\, l / a$	31.6±0.8	mΩ

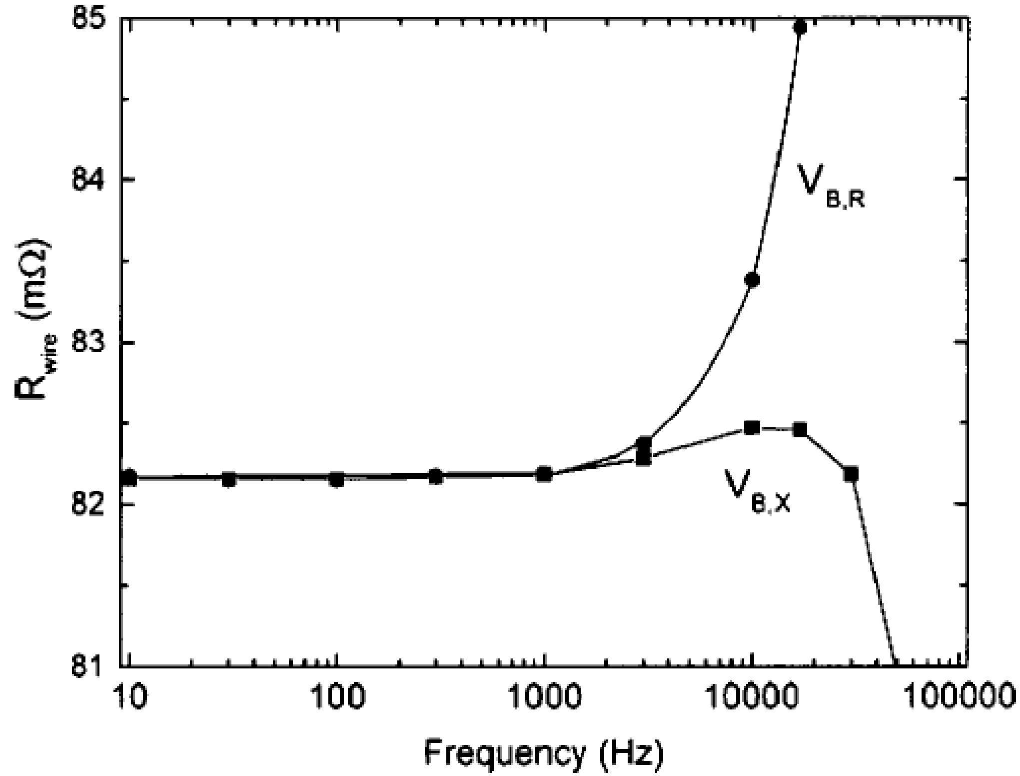
A basic lock-in amplifier experiment for the undergraduate laboratory

K. G. Libbrecht,[a)] E. D. Black, and C. M. Hirata
Norman Bridge Laboratory of Physics, California Institute of Technology 264-33, Pasadena, California 91125

(Received 9 August 2002; accepted 9 April 2003)

$$V_R^{\ 2} = V_x^{\ 2} + V_y^{\ 2}$$

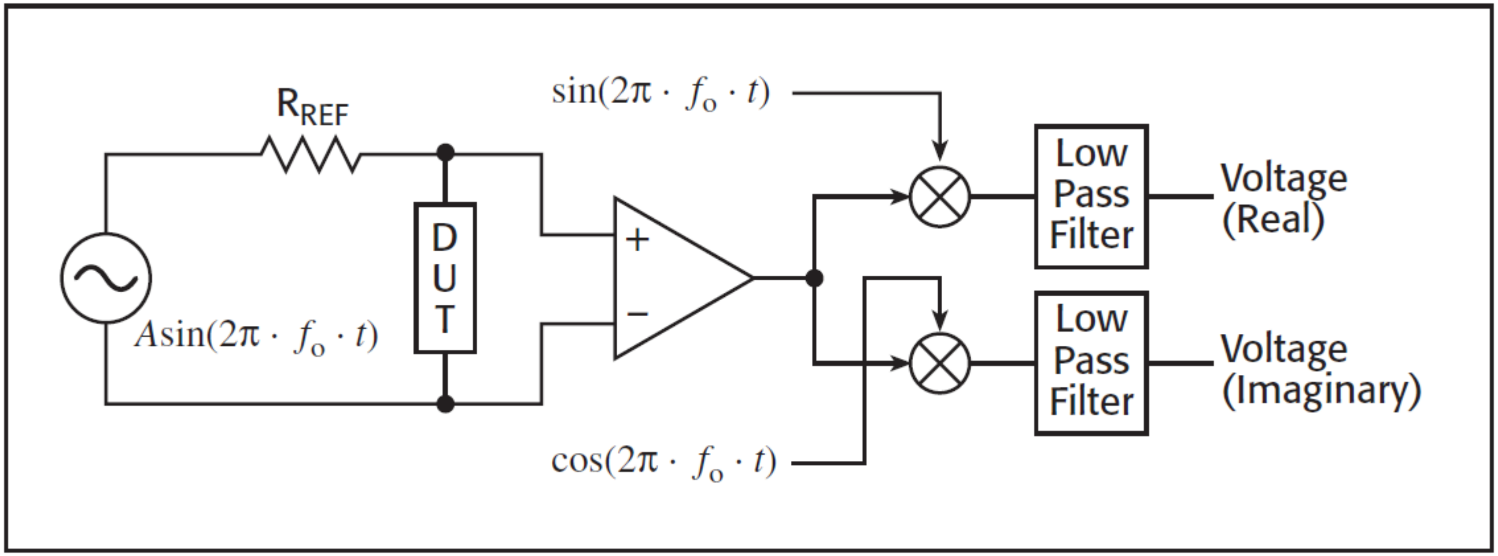
Appears to arise from inductive reactance



Suspect Claim 2:

Fig. 5. A measurement of R_{wire} as a function of the signal generator frequency with a large fixed input voltage. Specifically, $V_{A,\mathrm{set}}$ was a 1-V sine wave, and the lock-in time constant was 3 s. R_{wire} was determined using either the total signal amplitude $V_{B,R}$ or the in-phase component $V_{B,X}$. This graph demonstrates systematic effects in the measurement that arise from capacitive effects. These effects are reduced by using the in-phase signal $V_{B,X}$, but they are not eliminated.

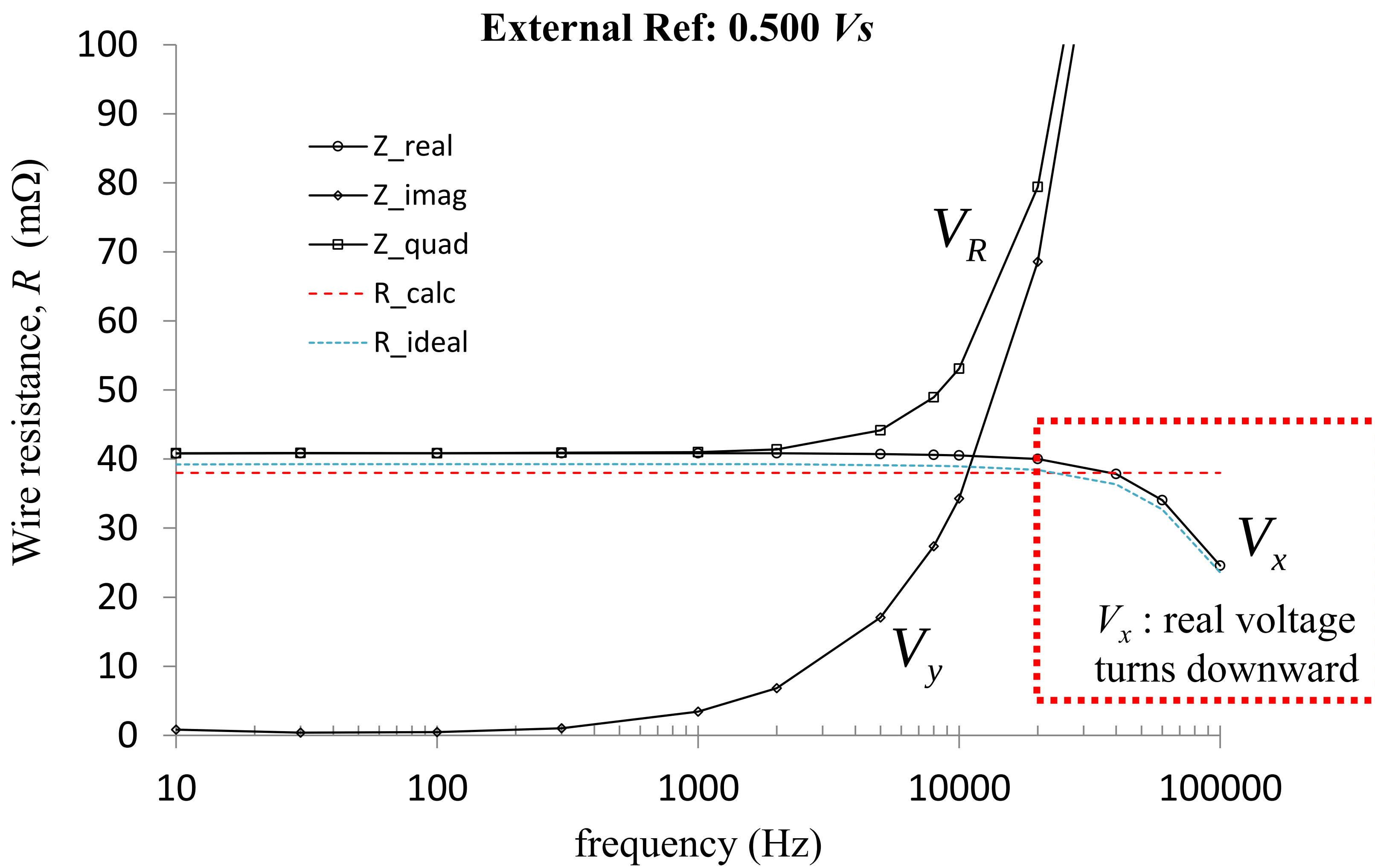
Simplified block diagram of a lock-in amplifier setup to measure the voltage of a DUT at low power.



The amplified voltage from the DUT is multiplied by both a sine and a cosine wave with the same frequency and phase as the applied source and then put through a low pass filter. In most cases, the multiplication and filtering is performed digitally within the lock-in amplifier after the DUT signal is digitized.

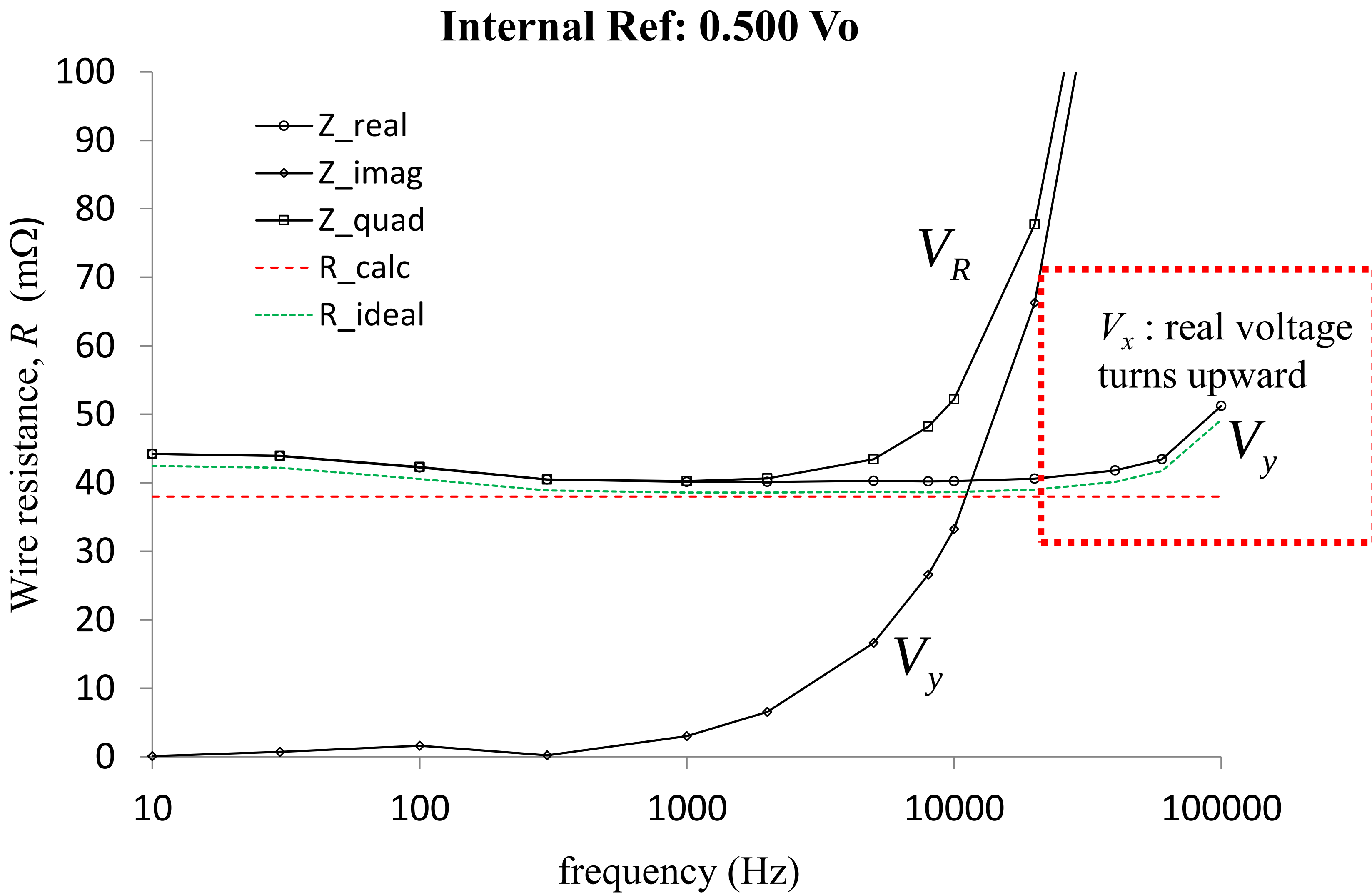
RESISTANCE IS FUTILE

$$V_R^{\ 2} = V_x^{\ 2} + V_y^{\ 2}$$



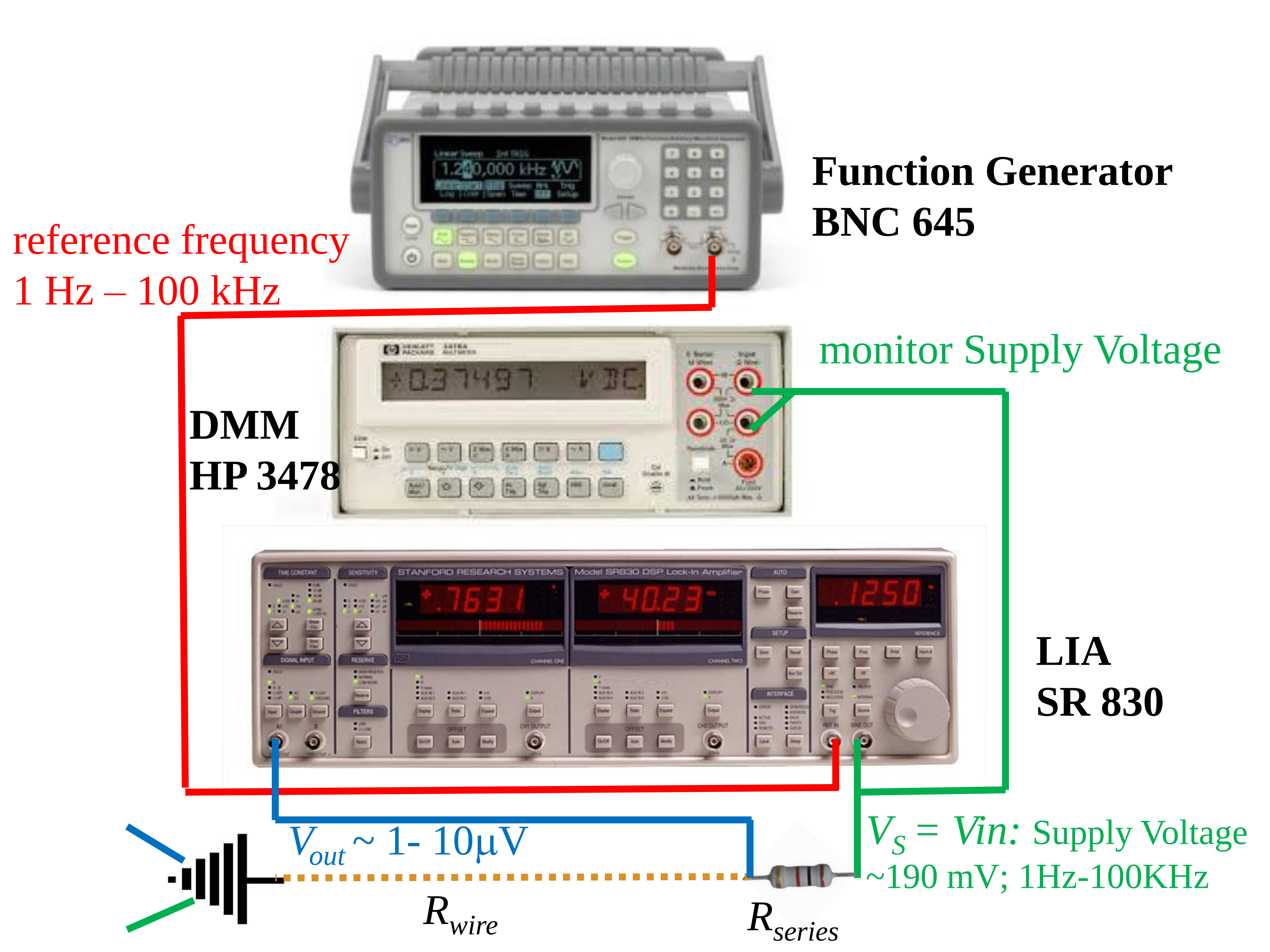
RESISTANCE IS FUTILE

$$V_R{}^2 = V_x{}^2 + V_y{}^2$$



….and hence the mixed internal/external method

--shown previously and in the next slide

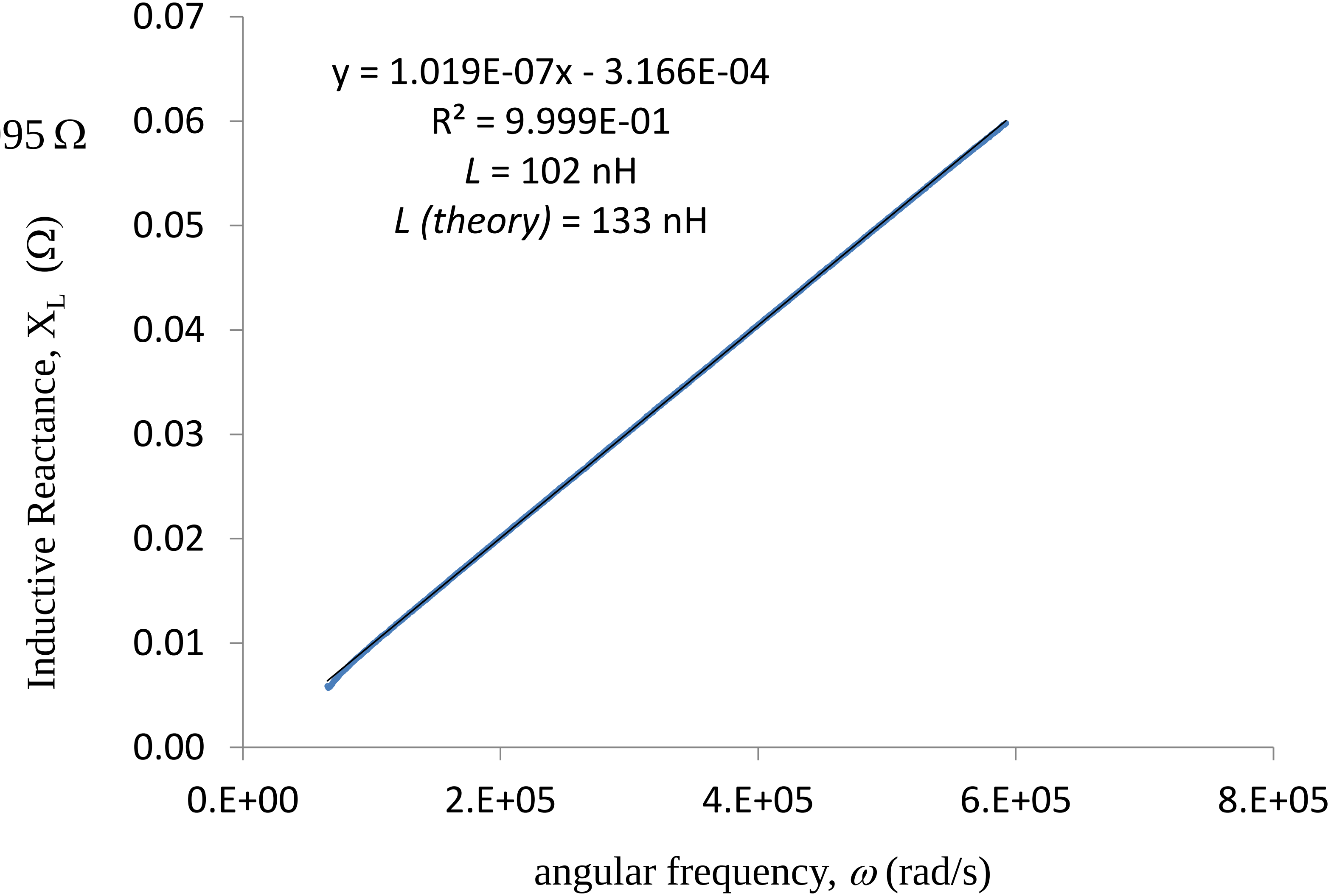


Function Generator
BNC 645
reference frequency
1 Hz – 100 kHz
DMM
HP 3478
monitor Supply Voltage
LIA
SR 830
$V_{out} \sim$ 1- 10μV
$V_S = Vin$: Supply Voltage
~190 mV; 1Hz-100KHz
R_{wire}
R_{series}

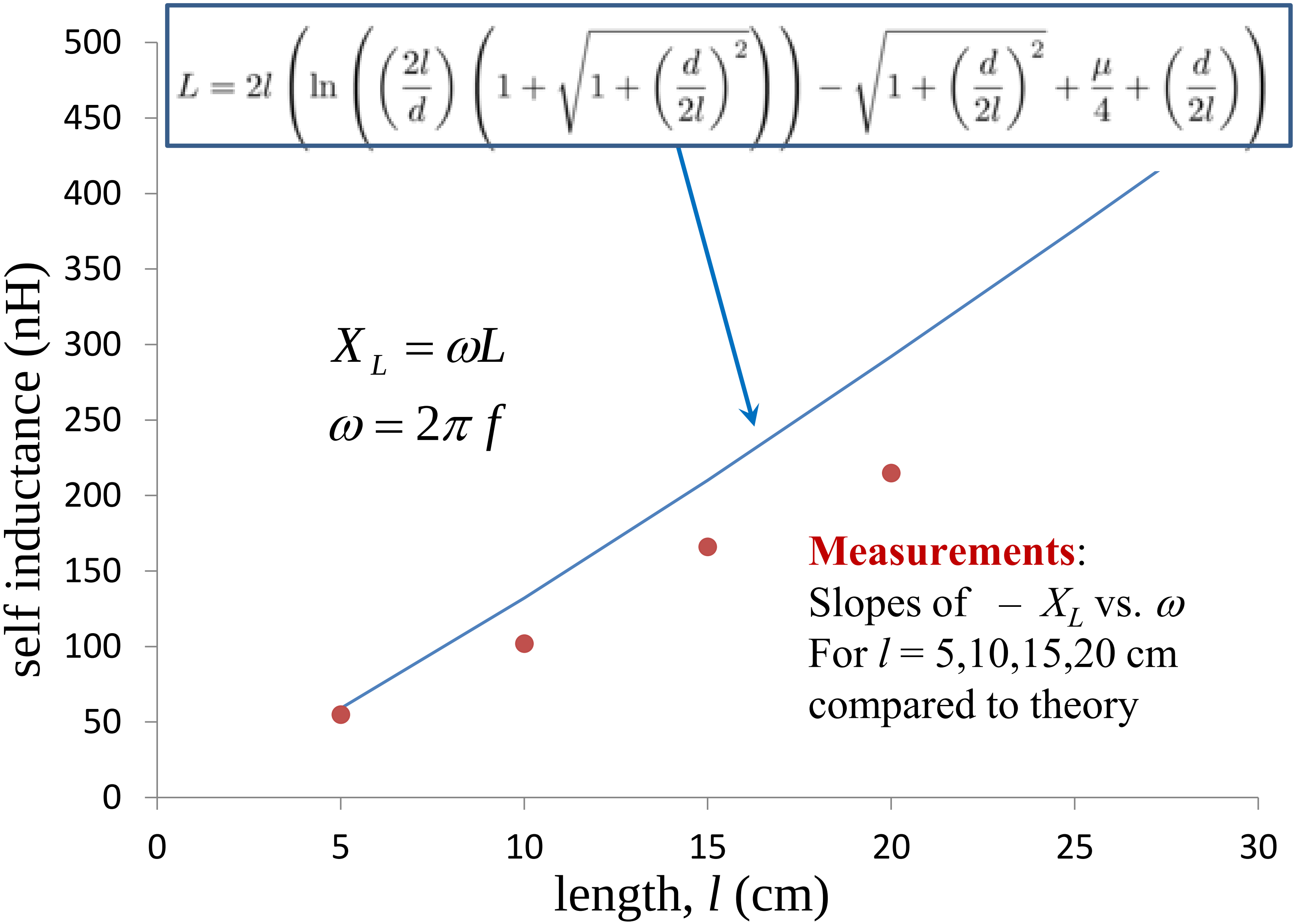
And for the inductance…

X_L (Ω) vs. ω (rad/s) : 10 cm wire (AWG 30)

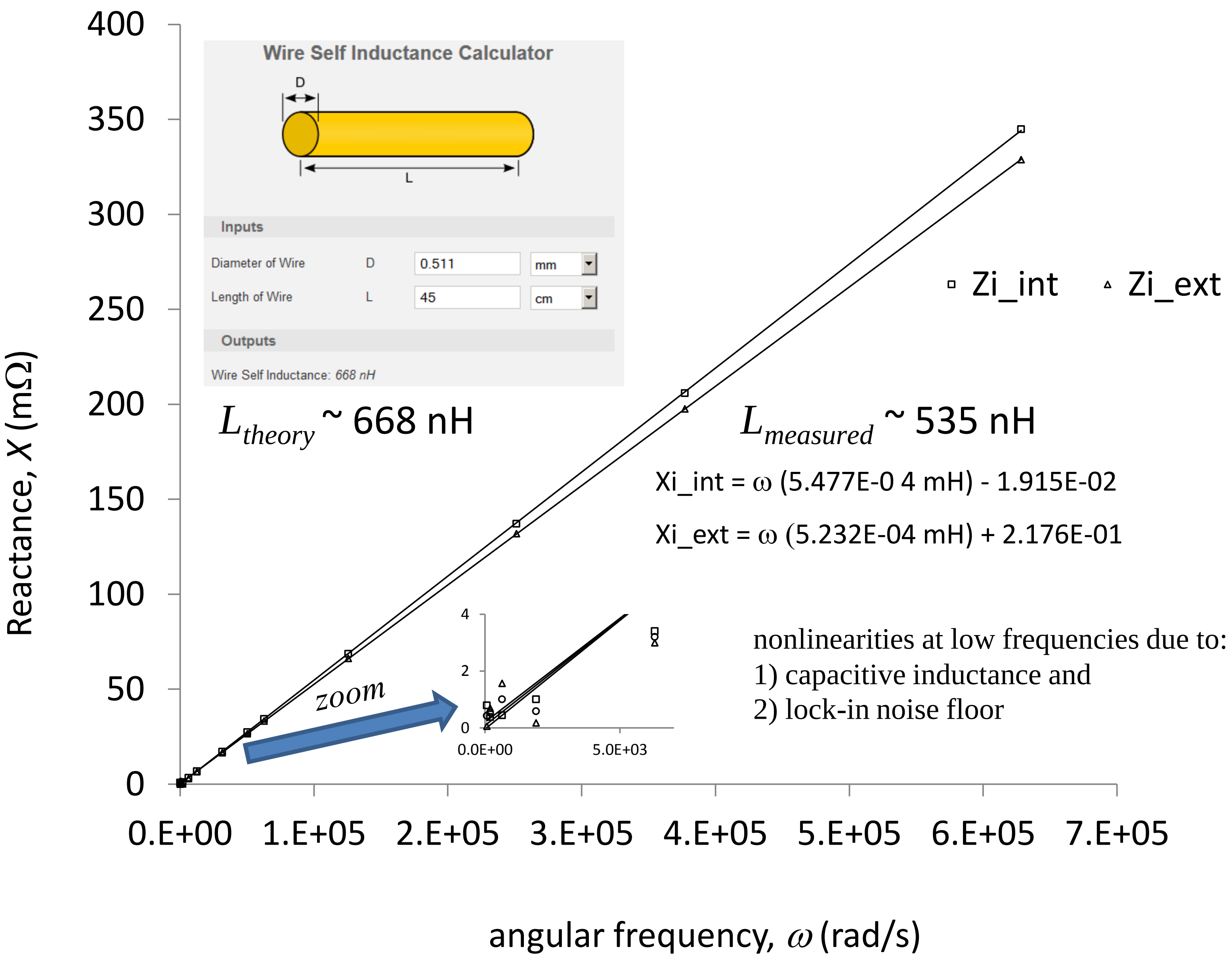
$$X_L = \frac{V_y}{V_{\text{supply}}} \times 995\,\Omega$$



The self inductance of a single wire in free space is defined below.



X_L (mΩ) vs. ω (rad/s) : 45.5 cm wire (AWG 24)



Conclusions

In spite of the Tupta (whitepaper) claim, a lock-in amplifier, when utilized effectively, will indeed provide 1% accuracy in the measurement of such a low resistance (R_{exp}~31mΩ) acquired within the low power limit (P_{exp}~30nW) ascribed therein.

The characterization of in-phase (real) and out-of-phase (imaginary) voltage drops across the resistive wire as functions of frequency show that inductive reactance, and NOT capacitive reactance, is indeed the major contributor to the complex impedance. This conclusion follows quite naturally from using the lock-in amplifier as a vector voltmeter, and is based primarily on elementary notions of impedance.